Higher School Certificate

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Term 1 (Set 1)
Question 11

(a) \[ \sqrt{\frac{234.6}{23.1 \times 3.8}} = 1.38774 \ldots \]
\[ = 1.39 \quad (3 \text{sf}) \]

(b) \[ 2x^2 - 7x - 15 \]
\[ = 2x^2 - 10x + 3x - 15 \]
\[ = 2x(x - 5) + 3(x - 5) \]
\[ = (x - 5)(2x + 3) \]

(c) \[ |2x - 1| > 5 \]
\[ 2x - 1 > 5 \quad \text{or} \quad 2x - 1 < -5 \]
\[ 2x > 6 \quad 2x < -4 \]
\[ x > 3 \quad x < -2 \]
\[ \therefore x > 3 \quad \text{or} \quad x < -2 \]

(d) \[ \frac{y^2 - 16}{2y - 8} = \frac{(y - 4)(y + 4)}{2(y - 4)} \]
\[ = \frac{y + 4}{2} \]

(e) \[ 2^{\frac{3x + 1}{7}} = 128 \]
\[ 2^{\frac{3x + 1}{7}} = 2^7 \]
\[ 3x + 1 = 7 \]
\[ \therefore x = 2 \]

(f) \[ 3 - 2x \leq 11 \]
\[ -2x \leq 8 \]
\[ x \geq -4 \]

(g) \[ \frac{6}{\sqrt{7} - \sqrt{5}} \cdot \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}} \]
\[ = \frac{6(\sqrt{7} + \sqrt{5})}{7 - 5} \]
\[ = 3(\sqrt{7} + \sqrt{5}) \]

(h) \[ x^2 = 6x \]
\[ x^2 - 6x = 0 \]
\[ x(x - 6) = 0 \]
\[ \therefore x = 0 \quad \text{or} \quad x = 6 \]
Term 1 (Set 1)
Question 12

(a) \[
\frac{4 \cdot 6 \times 5}{6 \cdot 3 + 5 \cdot 1} = 2.38070 \ldots
\]
   \[
   = 2.38 \quad (2 \text{ d.p.})
\]

(b) \[
\alpha + b \sqrt{10} = \frac{1}{\sqrt{10} - 3} \times \frac{\sqrt{10} + 3}{\sqrt{10} + 3}
\]
   \[
   = \frac{\sqrt{10} + 3}{10 - 9}
\]
   \[
   = 3 + \sqrt{10}
\]
   \[
   \therefore \alpha = 3 \quad \text{and} \quad b = 1
\]

(c) \[
|3x - 1| = 8
\]
   \[
   3x - 1 = 8 \quad \text{or} \quad 3x + 1 = 8
\]
   \[
   3x = 9 \quad \text{or} \quad 3x = 7
\]
   \[
   x = 3 \quad \text{or} \quad x = -\frac{7}{3}
\]

(d) \[
\frac{4x - 3}{x} = 7
\]
   \[
   4x - 3 = 7x
\]
   \[
   -3 = 3x
\]
   \[
   x = -1
\]

(e) \[
\frac{3}{n} + \frac{1}{n+1} = \frac{3(n+1) + n}{n(n+1)}
\]
   \[
   = \frac{3(n+1) + n}{n(n+1)}
\]
   \[
   = \frac{3n + 3 + n}{n(n+1)}
\]
   \[
   = \frac{4n + 3}{n(n+1)}
\]

(f) \[
(\sqrt{2} - 1)(3\sqrt{2} + 2)
\]
   \[
   = \sqrt{2}(3\sqrt{2} + 2) - 1(3\sqrt{2} + 2)
\]
   \[
   = 6 + 2\sqrt{2} - 3\sqrt{2} - 2
\]
   \[
   = 4 - \sqrt{2}
\]

(g) \[
y = 2x - 5
\]
   \[
x + 3y = 6
\]
   \[
\text{Sub } (1) \text{ into } (2)
\]
   \[
x + 3(2x - 5) = 6
\]
   \[
x + 6x - 15 = 6
\]
   \[
7x = 21
\]
   \[
x = 3
\]
   \[
\text{Sub } x = 3 \text{ into } (1)
\]
   \[
y = 6 - 5
\]
   \[
y = 1
\]
   \[
\therefore x = 3 \text{ and } y = 1
\]

(h) \[
2x^3 - 250 = 2(x^3 - 125)
\]
   \[
= 2(x^3 - 5^3)
\]
   \[
\text{Using}
\]
   \[
x^3 - y^3 = (x - y)(x^2 + xy + y^2)
\]
   \[
2x^3 - 250 = 2(x - 5)(x^2 + 5x + 25)
Term 1 (Set 1)
Question 13

(a) Let $A$, $B$ and $C$ be respectively the points $(5,2)$, $(-1,8)$ and $(p,6)$. If the gradient $AB = \text{gradient } BC$ then the points will be collinear and hence $p$ will be calculated.

$$m_{AB} = \frac{6-2}{-1-5} = -1, \quad m_{BC} = \frac{6-8}{p+1} = -\frac{2}{p+1}$$

$$\therefore -\frac{2}{p+1} = -1 \rightarrow 2 = p + 1 \rightarrow p = 1$$

(b) Rearrange $2x + 3y = 4$ and $2x - ay = 6$ to make $y$ the subject.

$$2x + 3y = 4 \rightarrow 3y = 4 - 2x \rightarrow y = \frac{4 - 2x}{3}$$

$$2x - ay = 6 \rightarrow ay = 2x - 6$$

$$y = \frac{x}{a} - \frac{6}{a} \rightarrow m_1 = \frac{2}{3}, \quad m_2 = \frac{1}{a}$$

For lines to be perpendicular $m_1 \cdot m_2 = -1$

$$\therefore \frac{2}{3} \cdot \frac{1}{a} = -1 \rightarrow -2 = -3a \rightarrow a = \frac{2}{3}$$

(c) Solving the equations simultaneously

$$3x - 2y = 11 \rightarrow (1)

2x + 3y = 16 \rightarrow (2)$$

1. $3x - 2y = 33 \rightarrow (3)$

2. $4x + 6y = 32 \rightarrow (4)$

3. $13x = 65 \rightarrow x = 5$

(d) Using the ‘k’ method

$$x + 3y - 6 + k(2x - y - 5) = 0 \rightarrow (1)$$

Substituting the co-ordinates of $(2,0)$

$$2 + 0 - 6 + k(4 - 0 - 5) = 0 \rightarrow 8 + k = 0 \rightarrow k = -8$$

Sub $k = -8$ into $(1)$

$$x + 3y - 6 - 8x + 4y + 20 = 0 \rightarrow -7x + 7y + 14 = 0$$

$$\therefore x - y - 2 = 0$$

(e) The perpendicular distance from the centre of the circle to the line is the radius $r$.

Using the perpendicular distance formula from the origin $(0,0)$ and the line $3x - 4y + 5 = 0$.

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|13 \times 0 + 4 \times 0 + 5|}{\sqrt{3^2 + (-4)^2}} = 1$$

$$\therefore r = 1$$
Term 1 (Set 1)
Question 14

(a) (i) \[ AB = \sqrt{(0-5)^2 + (4-8)^2} = 5 \]

(ii) \[ AB = BC \rightarrow C(8,8) \]

(iii) gradient of \( AB = \frac{4-8}{0-5} = \frac{-4}{5} \)
Now use \( C(8,8) \) and \( m_{AB} = \frac{4}{5} \) to find the equation of \( EC \).
\[ y - 8 = \frac{4}{5}(x - 8) \rightarrow 3y - 24 = 4x - 32 \]
\[ 4x - 3y - 8 = 0 \]

(iv) gradient of \( AC \) \( m_{AC} = \frac{8 - y}{8 - 0} = \frac{1}{2} \)
The co-ordinates of \( D \) are \( (5, k) \)
gradient of \( BD \) \( m_{BD} = \frac{8 - y}{3 - 5} = \frac{1}{2} \)
\[ m_{AC} \times m_{BD} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \]
\[ \therefore AC \perp BD \]

(v) Area of \( ABCE \) = Area \( ABCD + \) Area \( \triangle AED \)
\[ ABCD = 5 \times 4 = 20 \text{ } \text{u}^2 \]
Co-ordinates of \( E \) when \( x = 0 \)
\[ -3y - 8 = 0 \rightarrow y = \frac{8}{3} \]
\[ \therefore E \left( 0, \frac{8}{3} \right) \]
\[ \triangle AED = \frac{1}{2} \times 5 \times \left( \frac{8}{3} + 4 \right) = \frac{50}{3} \text{ } \text{u}^2 \]
Area of \( ABCE \) = \[ 20 + \frac{50}{3} \]
\[ = \frac{110}{3} \text{ } \text{u}^2 \]

(b) \[ 5x + 12y - 5 = 0 \] \( \rightarrow \)
Let \( y = 0 \) in (i) \( \rightarrow 5x = 5 \)
\( x = 1 \)
\( \therefore (1,0) \) lies on (i)
Now find the perpendicular distance from \( (1,0) \) to \( 5x + 12y + 8 = 0 \)
\[ p = \frac{|5 \times 1 + 12 \times 0 + 8|}{\sqrt{5^2 + 12^2}} \]
\[ \therefore p = 1 \]

(c) Draw each line:
\[ x + y = 1, y - x = 1, \]
\[ x + y = -1 \text{ and } y - x = -1 \]
Now test the point \( (0,0) \) in each of the inequalities
\[ x + y \leq 1, \text{ } y - x \leq 1, \]
\[ x + y \geq -1 \text{ and } y - x \geq -1. \]
If \( (0,0) \) ‘satisfies’ the particular region then shade that side of the line.
Hence
Term 1 (Set 1)
Question 15

(a) (i)

(ii) $f(-2) = 1$
$f(1) = -1$
$f(2) = -1$

$\therefore f(-2) + f(1) - f(2) = 1$

(b) (i)

(ii) 
Area of circle = $\pi \times r^2$

$= 4 \pi$

Area of sector = $\frac{1}{4} \times 4 \pi$

$\therefore R = \pi \times a^2$

(c) (i)

$$g(x) = \frac{1 - \frac{a}{h}}{\frac{a+h}{a(h)} \frac{a}{h}}$$

$$= \left[ \frac{a}{a(a+h)} - \frac{a+h}{a(a+h)} \right] \frac{1}{h}$$

$$= \frac{-h}{a(a+h)} \times \frac{1}{h}$$

$$= \frac{1}{a(a+h)}$$

(ii) $g(0) = \frac{-1}{a^2}$

(d) (i) $f(x) = \sqrt{x^2 - 4}$

$x^2 - 4 \geq 0 \Rightarrow (x-2)(x+2) \geq 0$

$\therefore$ Domain is $x \leq -2$ or $x \geq 2$

(ii) For the positive square root

Range is $f(x) \geq 0$
Term 1 (Set 1)
Question 16

(a) (i)

Let $P(x, y)$ be a variable point that is equidistant from both lines and let $K$ be that distance.

(i) $K = \frac{|3x - 4y + 5|}{\sqrt{9 + 16}}$

$$K = \frac{13x - 4y + 5}{5}$$

(ii) $K = \frac{|3x - 4y + 9|}{\sqrt{9 + 16}}$

$$K = \frac{13x - 4y + 9}{5}$$

(iii) $13x - 4y + 5 = -3x + 4y + 9$

(iv) $3x - 4y + 5 = 0$ is the locus

Note: $3x - 4y + 5 = 3x - 4y + 9$

will give an invalid solution.

(b)

The point $P(x, y)$ moves along the perpendicular bisector of $AB$.

The mid-point $M$ is $\left(-\frac{1 + 3}{2}, \frac{2 + 4}{2}\right)$

i.e. $M(1, 3)$

The gradient of $AB = \frac{4 - 2}{3 - 1} = \frac{1}{2}$

The gradient of the perpendicular bisector = $-2$

The locus equation is

$$y - 3 = -2(x - 1)$$

$$y - 3 = -2x + 2$$

$$2x + y - 5 = 0$$

(i) Centre $(-2, 1)$

(ii) radius = 2 units
Term 1 (Set 2)
Question 11

(a) \( \sqrt{\pi^3 + 4} = 5 \cdot 9166 \approx 5 \cdot 9 \) (2 significant figures)

(b) \( \sin \theta \quad a^3 + b^3 = (a+b)(a^2 - ab + b^2) \)
\[ 8x^3 + 27 = (2x)^3 + 3^3 \]
\[ = (2x+3)(4x^2 - 6x + 9) \]

(c) \[ |1 - 2x| \leq 7 \quad \Rightarrow \quad |2x - 1| \leq 7 \]
\[ -7 \leq 2x - 1 \leq 7 \]
\[ -6 \leq 2x \leq 8 \]
\[ -3 \leq x \leq 4 \]

(d) \[ \frac{x^2 + 2x + 1}{x^2 - 1} = \frac{(x+1)^2}{(x+1)(x-1)} = \frac{x+1}{x-1} \]

(e) \[ 6^{4-y} \times 2^y \div 3^{3-y} \]
\[ = (2 \times 3)^{4-y} \times 2^y \div 3^{3-y} \]
\[ = 2^{4-y} \times 3^{4-y} \times 2^y \div 3^{3-y} \]
\[ = 2^4 \times 3 \]
\[ = 48 \]

(f) \[ 4(x - 3) \leq 2 - 2(x - 1) \]
\[ 4x - 20 \leq -2x + 2 \]
\[ 6x - 22 \leq 0 \]
\[ x \leq \frac{11}{3} \]

(g) \[ \frac{1}{1+\sqrt{2}} - \frac{1}{1-\sqrt{2}} \]
\[ = \frac{1 - \sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} - \frac{1 + \sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} \]
\[ = \frac{-2\sqrt{2}}{1-2} = -2\sqrt{2} \]

(h) \[ x - \frac{2}{x} = 1 \]
\[ x^2 - 2 = x \]
\[ x^2 - x - 2 = 0 \]
\[ (x-2)(x+1) = 0 \]
\[ \therefore x = 2 \quad \text{or} \quad x = -1 \]
Term 1 (Set 2)

Question 12

(a) \[
\frac{(3.25)^2}{5.98 - 3.67} = 4.5725\ldots
\]
\[= 4.57 \quad (2 \text{ d.p.})
\]

(b) \[5x - 3(x - 4) = 5x - 3x + 12
\]
\[= 2x + 12
\]

(c) \[|x - 1| = x + 3 \quad (*)
\]

(i) \[x - 1 = x + 3 \quad \text{or} \quad 1 - x = x + 3
\]
   (i) No Solution
   (ii) \[-2x = 2
\]
   \[x = -1
\]

Check in \((*)\) \[\text{RHS} = -1 + 3 = 2
\]

\[\therefore \text{RHS} > 0
\]
\[\therefore \text{Only solution is} \quad x = -1
\]

(d) \[\frac{x}{2} - \frac{x + 1}{3} = 1
\]
\[\frac{3x}{6} - \frac{2(x + 1)}{6} = 1
\]
\[\frac{3x - 2(x + 1)}{6} = 1
\]
\[3x - 2x - 2 = 6
\]
\[x = 8
\]

(e) \[A = 2\pi rh + 2\pi r^2
\]
\[A - 2\pi r^2 = 2\pi rh
\]
\[h = \frac{A - 2\pi r^2}{2\pi r}
\]
\[h = \frac{120 - 2\pi (4.25)^2}{2\pi \times 4.25}
\]
\[\therefore h = 0.24
\]

(f) \[\frac{\sqrt{5} + 1}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1}
\]
\[= \frac{5 + 2\sqrt{5} + 1}{4}
\]
\[= \frac{6 + 2\sqrt{5}}{4}
\]
\[= \frac{3 + \sqrt{5}}{2}
\]

(g) \[y = x^2 - 1
\]
\[x + y - 6 = 0 \quad (ii)
\]

\[\therefore x + x^2 - 6 = 0 \quad (ii)
\]
\[\text{Sub } y = x^2 \quad (i)
\]
\[x + 6 = 0
\]
\[x = -6
\]
\[x = 3, \quad y = 9
\]

(h) \[x^2 - y^2 + x + y
\]
\[= (x+y)(x-y) + 1(x+y)
\]
\[= (x+y)(x-y+1)
\]
Term 1 (Set 2)
Question 13

(a) Let B have the coordinates \((x, y)\)

Since \(A(-2, 8)\) and \(M(6, 2)\) then

\[
\frac{x - (-2)}{2} = \frac{6}{2} \quad \Rightarrow \quad x = 14
\]

\[
\frac{y - 8}{2} = 2 \quad \Rightarrow \quad y = 4
\]

\[\therefore B(14, 4)\]

(b) \[2x + 3y = 4 \quad \text{-- (i)}\]

The line perpendicular to (i) must be of the form

\[3x - 2y = k \quad \text{-- (ii)}\]

Now \((-1, 3)\) lies on (ii)

\[3(-1) - 2(3) = k \quad \Rightarrow \quad k = -9\]

\[\therefore 3x - 2y = -9\]

or in general form \(3x - 2y + 9 = 0\)

(c) \[
\begin{array}{c}
\text{y} \\
\hline
\text{x} = 1 \\
(0, 1) \\
(1, 0)
\end{array}
\]

\[
\text{From the diagram above, the line } x + y = 1 \text{ makes an angle of } 45^\circ \text{ with the negative direction of the } x\text{-axis. Hence the angle between this line and the line } x = 1 \text{ is } 45^\circ.
\]

(d) \[x + y = 6 \quad \text{-- (i)}\]

\[2x - y = 4 \quad \text{-- (ii)}\]

\[(i) + (ii) \quad 3x = 10 \quad \Rightarrow \quad x = \frac{10}{3}\]

\[\therefore \frac{10}{3} + y = 6 \quad \Rightarrow \quad y = \frac{8}{3}\]

\[\therefore \text{Point of intersection is } (\frac{10}{3}, \frac{8}{3})\]

\[\text{Gradient is 2}\]

\[y - \frac{8}{3} = 2(x - \frac{10}{3})\]

\[\frac{3y - 8}{3} = 2(3x - 10)\]

\[3y - 8 = 6x - 20\]

\[2x - y - 6 = 0\]

(e) \[y = 2x + 1, \text{ now change to the general form}\]

\[2x - y + 1 = 0\]

Using the perpendicular distance formula from a point to a line formula

\[d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}\]

\[a = 2, \quad b = -1, \quad c = 1\]

\[x_1 = 2, \quad y_1 = -3\]

\[d = \frac{|2 \times 2 + 3 \times 1 + 1|}{\sqrt{4 + 1}}\]

\[d = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}\]

\[= \frac{3}{\sqrt{5}}\]
(a) 

(i) \( OB = \sqrt{(8-0)^2 + (8-0)^2} \) 
\[ = 8\sqrt{2} \]

(ii) Let \( M \) be the midpoint of \( OB \) 
\[ M \left( \frac{8+0}{2}, \frac{8+0}{2} \right) = M(4,4) \]

(iii) Gradient of \( AC \), \( m_{AC} = \frac{6-2}{2-6} = -1 \)

(iv) Gradient of \( OB \), \( m_{OB} = \frac{8-0}{8-0} = 1 \)  
\[ \therefore m_{AC} \times m_{OB} = -1 \times 1 \]  
\[ = -1 \]  
\[ \therefore AC \perp OB \]

(v) Let \( N \) be the midpoint of \( AC \).  
\[ N \left( \frac{2+6}{2}, \frac{6+2}{2} \right) = N(4,4) \]  
\[ \therefore \text{Midpoint of } AC = \text{Midpoint of } OB \]  
\[ \therefore AC \text{ bisects } OB \]  
\[ \text{and } OB \text{ bisects } AB \]  
\[ \text{Now } AC \perp OB \]  
\[ \therefore \text{The diagonals of } OABC \text{ bisect each other at right angles.} \]  
\[ \therefore OABC \text{ is a rhombus.} \]

(vi) \[ AC = \sqrt{(2-6)^2 + (6-2)^2} \]  
\[ = \sqrt{16+16} \]  
\[ = 4\sqrt{2} \]  
\[ \text{Area } = \frac{1}{2} \times 4\sqrt{2} \times 8\sqrt{2} \]  
\[ = 32 \text{ u}^2 \]

(b) The radius of \( x^2 + y^2 = 16 \) is 4. If \( 4x + 3y - 20 = 0 \) is a tangent to this circle then the perpendicular distance from the centre \((0,0)\) of the circle to this line will be 4 units.
\[ d = \frac{|4\times0 + 3\times0 - 20|}{\sqrt{4^2 + 3^2}} \]  
\[ = 4 \]  
\[ \therefore 4x - 3y - 20 \text{ is a tangent to the circle.} \]

(c) \[ |x + y| \leq 1 \]
\[ \therefore x + y \leq 1 \text{ or } -(x+y) \leq 1 \]
\[ \therefore x + y \leq 1 \text{ or } x + y \geq -1 \]

Draw the lines \( x + y = 1 \) and \( x + y = -1 \) use \((0,0)\) to test which side of the lines to shade for the corresponding inequalities.
Term 1 (Set 2)

Question 15

(a) (i) \( f(-1) = 2^{-1} = \frac{1}{2} \)
    \( f(1) = a \)
    \( f(5) = \frac{5}{b} \)
    \[ f(-1) = f(1) \Rightarrow a = \frac{1}{2} \]
    \[ f(-1) = f(5) \Rightarrow \frac{5}{b} = \frac{1}{2} \]
    \[ \therefore a = \frac{1}{2} \text{ and } b = 10 \]

(ii) \( f(-2) = 2^{-2} = \frac{1}{4} \)
    \( f(1) = \frac{1}{2} \)
    \( f(5) = \frac{1}{2} \)
    \[ f(-2) + f(1) - f(5) = \frac{1}{4} - \frac{1}{2} = \frac{1}{4} \]

(b) Sketch the functions \( y = x^2 \)
    and \( y = 1 - |x| \).
    Select a point that does not lie on either of the functions and test this point in the corresponding regions (e.g., \( (0, \frac{1}{2}) \)). Then shade the area.

(c) (i) \( q(x) = \frac{(x^2 + x) - (4+2)}{x-2} \)
    \[ = \frac{x^2 + x - 6}{x-2} \]
    \[ = \frac{(x-2)(x+3)}{x-2} \]
    \[ = x + 3 \]

(ii) \( q(2.01) = 2.01 + 3 \)
    \[ = 5.01 \]

(d) (i) \( f(x) = \sqrt{25 - x^2} \) is a semi-circle with a radius of 5 units.
    The domain is \(-5 \leq x \leq 5\)

(ii) Since \( \sqrt{25 - x^2} \geq 0 \) and the radius is 5 units then the range is \( 0 \leq f(x) \leq 5 \)
Term 1 (Set 2)

Question 16

(a) (i) \( f(x) = x^3 - x \)

Let \( x = a \rightarrow f(a) = a^3 - a \)

Let \( x = -a \rightarrow f(-a) = (-a)^3 - (-a) \)

\[ = -a^3 + a \]

\[ = -(a^3 - a) \]

\[ = -f(a) \]

\[ \\Rightarrow f(-a) = -f(a) \]

\[ \therefore f(x) \text{ is odd} \]

(ii)

(b) (i)

Show that \( \frac{dx}{dx-1} = 1 + \frac{1}{x-1} \)

R.H.S. = \( \frac{x-1 + 1}{x-1} \)

\[ = \frac{x-1 + 1}{x-1} \]

\[ = \frac{x}{x-1} \]

\[ = \text{L.H.S.} \]

\[ \therefore \text{R.H.S. = L.H.S} \]

(ii)

This locus is a circle with centre \((-1,2)\) and radius 2.
The general form is \( (x - h)^2 + (y - k)^2 = r^2 \)

\[ \therefore \text{Locus is } (x+1)^2 + (y-2)^2 = 4 \]

(d) (i)

\[ PA = \sqrt{(x+3)^2 + y^2} \]

\[ PB = \sqrt{(x-3)^2 + y^2} \]

\[ \therefore \sqrt{(x+3)^2 + y^2} = 2 \sqrt{(x-3)^2 + y^2} \]

\[ (x+3)^2 + y^2 = 4 (x-3)^2 + 4y^2 \]

\[ x^2 + 6x + 9 + y^2 = 4(x^2 - 6x + 9 + y^2) \]

\[ = 4x^2 - 24x + 36 + 4y^2 \]

\[ 0 = 3x^2 - 30x + 27 + 3y^2 \]

\[ x^2 - 10x + y^2 + 9 = 0 \]

\[ x^2 - 10x + 25 + y^2 = 25 - 9 \]

\[ (x-5)^2 + y^2 = 16 \]

(ii)

This is a circle with centre \((5,0)\) and radius 4 units.
Term 1 (Set 3)
Question 11
(a) \[12,756 \text{ km} \]
Round 2 up to 3
\[13,000 \text{ km} \text{ to 2 sig. figs} \]
(b) \[x^3 + x^2 - 2x - 1 \]
\[x^2(x+1) - 1(x+1) \]
\[(x+1)(x^2-1) \]
\[(x+1)(x+1)(x-1) \]
\[(x+1)^3(x-1) \]
(c) \[|2x+1| = 1-x \]
\[2x+1 = 1-x \text{ or } -2x-1 = 1-x \]
\[3x = 0 \quad \text{or} \quad -x = 2 \]
\[x = 0 \quad \text{or} \quad x = -2 \]
Test \( x = 0 \)
Test \( x = -2 \)
R.H.S. = 1 - 0 = 1
R.H.S. = 1 - (-2) = 3
\[\because x = 0 \text{ and } x = 3 \text{ are solutions} \]
(d) \[m^3 - 8 = \frac{m^3 - 2^3}{m^2 + 2m + 4} \]
\[= \frac{(m-2)(m^2 + 2m + 4)}{m^2 + 2m + 4} \]
\[= m - 2 \]
(e) \[\frac{5^n - 5^4}{4} = \frac{5^n(5-1)}{4} = \frac{5^n}{4} \]
(f) \[\frac{x+1}{2} + \frac{x+2}{3} \leq 7 \]
\[\frac{3(x+1) + 2(x+2)}{6} \leq 7 \]
\[3x + 3 + 2x + 4 \leq 42 \]
\[5x + 7 \leq 42 \]
\[5x \leq 35 \]
\[x \leq 7 \]
(g) \[(2\sqrt{2} + 1)^2 - (2\sqrt{2} - 1)^2 \]
\[= (2\sqrt{2} + 1 + 2\sqrt{2} - 1)(2\sqrt{2} + 1 - 2\sqrt{2} + 1) \]
\[= 4\sqrt{2} \times 2 \]
\[= 8\sqrt{2} \]
(h) \[x^2 = 2x + 1 \]
\[x^2 - 2x + 1 = 1 + 1 \]
\[(x-1)^2 = 2 \]
\[x = 1 \pm \sqrt{2} \]
Term 1 (Set 3)  
Question 12

(a) \[ \sqrt[3]{\frac{987}{2\pi}} = 5.3956 \ldots \]
\[ = 5.40 \text{ (2 d.p.)} \]

(b) \[ \frac{1}{n} - \frac{1}{n+1} + \frac{1}{(n+1)^2} \]
\[ = \frac{(n+1)^2}{n(n+1)^2} - \frac{n(n+1)}{n(n+1)^2} + \frac{n}{n(n+1)^2} \]
\[ = \frac{n^2+2n+1-n^2-n}{n(n+1)^2} \]
\[ = \frac{2n+1}{n(n+1)^2} \]

(c) \[ |x-1| = |2x+1| \]
\[ x-1 = 2x+1 \text{ or } 1-x = 2x+1 \]
\[ -x = 2 \]
\[ -3x = 0 \]
\[ x = -2 \]
\[ x = 0 \]
\[ \therefore x = -2 \text{ or } x = 0 \]

(d) \[ \frac{2x-1}{x+1} = 2x \]
\[ 2x^2 = (2x-1)(x+1) \]
\[ = 2x^2 + x - 1 \]
\[ -x = -1 \]
\[ \therefore x = 1 \]

(e) \[ \frac{125}{150} \times \frac{x}{1.25} = 30000 \]
\[ x = \frac{30000}{1.25} \]
\[ = 24000 \]

(f) \[ -2 \leq 1-x < 3 \]
\[ -2 \leq 1-x \]
\[ 1-x < 3 \]
\[ -2 \leq 1-x \]
\[ -x < 2 \]
\[ x \leq 3 \]
\[ x > -2 \]
\[ \therefore -2 \leq x \leq 3 \]

(g) \[ \frac{|x|}{x} = \frac{x}{x} \text{ if } x > 0 \]
\[ \frac{|x|}{x} = 1 \]
\[ \frac{|x|}{x} = -\frac{x}{x} \text{ if } x < 0 \]
\[ \frac{|x|}{x} = -1 \]

(h) \[ \sqrt{63} + \sqrt{112} \]
\[ = \sqrt{9 \times 7} + \sqrt{16 \times 7} \]
\[ = 3\sqrt{7} + 4\sqrt{7} \]
\[ = 7\sqrt{7} \]
\[ \therefore a = 7 \]
Term 1 (Set 3)

Question 13

(a) 

\[ A(0,6) \]

\[ B \]

\[ C(0,-4) \]

\[ y = -2x + 6 \]

\[ y = 3x - 4 \]

To find \( M \) : 

\[ 2x + y = 6 \quad \text{(1)} \]

\[ 3x - y = 4 \quad \text{(2)} \]

\[ 5x = 10 \]

\[ x = 2, \quad y = 2 \]

In \( \triangle AMC \) 

\[ AC = 10, \quad MN = 2 \]

\[ \therefore \text{Area} = \frac{1}{2} \times 10 \times 2 = 10 \text{ units}^2 \]

(b) The line parallel to 

\[ 3x + 5y - 4 = 0 \] has the form 

\[ 3x + 5y - \kappa = 0 \quad (\star) \]

(2-1) lies on \((\star)\) 

\[ 3x + 5y - 1 - \kappa = 0 \]

\[ 1 - \kappa = 0 \]

\[ \therefore \kappa = 1 \]

\[ \therefore \text{Equation is} \quad 3x + 5y - 1 = 0 \]

(c) 

(i) Consider the gradients of each line.

\[ M_{AB} = \frac{1+2}{7-3} = \frac{3}{4} \]

\[ M_{BC} = \frac{9-1}{1-7} = \frac{8}{-6} = -\frac{4}{3} \]

\[ M_{AC} = \frac{9+2}{1-3} = -\frac{11}{2} \]

\[ \therefore M_{AB} \times M_{BC} = \frac{3}{4} \times -\frac{4}{3} = -1 \]

\[ \therefore AB \perp BC \]

(ii) 

\[ AB = \sqrt{(3-7)^2 + (-2-1)^2} = 5 \]

\[ BC = \sqrt{(7-1)^2 + (1-4)^2} = 10 \]

\[ \therefore \text{Area} = \frac{1}{2} \times 10 \times 5 = 25 \text{ units}^2 \]

(d) 

\[ \text{D}(-3,3) \]

\[ \text{B}(3,3) \]

\[ \text{C}(6,5) \]

\[ \text{D}(5,-1) \]

\[ x - 2y - 7 = 0 \]

Test \( D \) 

\[ \text{LHS} = 5 + 2 - 7 = 0 \]

\[ \therefore \text{LHS} = \text{RHS} \]

\[ \therefore \text{D}(5,-1) \]

(Also \( D'(3,-3) \) but not in cyclic order)
Term 1 (Set 3)

Question 14

(a) [Diagram of a triangle with points A(2,6), L, M, and C, and points O, N, B(4,0).

(i) \[ M_1 \left( \frac{2+4}{2}, \frac{6+0}{2} \right) \Rightarrow M_1(3,3) \]

(ii) \[ M_{om} = \frac{3-0}{3-0} = 1 \Rightarrow y = x \]

\[ N(2,0), A(2,6) \Rightarrow x = 2 \]

(iii) \[ x = 2 \Rightarrow y = 2 \]

\[ \therefore C(2,2) \]

(iv) \[ OC = \sqrt{(2-0)^2 + (2-0)^2} \]

\[ = \sqrt{8} \]

\[ = 2\sqrt{2} \]

(v) \[ OC = 2\sqrt{2} \]

\[ CM = \sqrt{2} \]

\[ \therefore OC:CM = 2\sqrt{2} : \sqrt{2} \]

\[ = 2 : 1 \]

(b) [Diagram of a triangle with a shaded area.

(vi) \[ L \left( \frac{2+0}{2}, \frac{6+0}{2} \right) \Rightarrow L(1,3) \]

\[ M_{LC} = \frac{3-2}{1-2} = \frac{1}{-1} = -1 \]

\[ M_{CB} = \frac{2-0}{2-4} = \frac{2}{-2} = -1 \]

\[ \therefore M_{LC} = M_{CB} \]

\[ \therefore L, C \text{ and } B \text{ are collinear} \]

(b) Draw each line:

\[ 2x + y - 2 = 0, 2x - y + 2 = 0 \]

Test a point (e.g. (0,1)) and shade the area that satisfies the inequality.
Term 1 (Set 3)

Question 15

(a)

(i)

(ii) \( f(-2) = 1, \ f(2) = -3 \)

\[
2f(-2) + f(2) = 2 \times 1 + (-3) = -1
\]

(b) (i)

(ii) \( \text{Area} = \frac{1}{2} \pi \times (1)^2 = \frac{\pi}{2} \)

(c) (i) \( g(x) = \frac{1}{x} - 1 \)

\[
= \frac{1 - x}{x} = -1 \quad \text{for} \quad x - 1
\]

\[
= - (x - 1) \quad \frac{1}{x - 1}
\]

\[
= -\frac{1}{x}
\]

(ii) \( g(1.01) = \frac{-1}{1.01} = -100 \quad \text{or} \quad 100 \)

(d)

(i) \( f(x) = \frac{1}{1 + x^2} \)

No restrictions on \( x \)

\( \therefore \) Domain is all real \( x \)

(ii) \( 1 + x^2 > 0 \) and \( \frac{1}{1 + x^2} \neq 0 \)

\( \therefore \) \( f(x) > 0 \)

\( f(x) = \frac{1}{1 + x^2} \) when \( x = 0 \) \( f(0) = 1 \)

which is the maximum value

\( \therefore \) Range is \( 0 < f(x) \leq 1 \)
Term 1 (Set 3)
Question 16

(a) (i) \( f(x) = x^4 + 1 \)
\( f(a) = a^4 + 1 \)
\( f(-a) = (-a)^4 + 1 \)
\( = a^4 + 1 \)
\[ \therefore f(-a) = f(a) \]
\[ \therefore f(x) \text{ is an even function} \]

(ii) \((-2, 15) \quad (2, 15)\)

(b) (i) \( A(-1, 2), \quad B(3, 6), \quad P(x, y) \)
\[ m_{PA} = \frac{y - 2}{x + 1}, \quad m_{PB} = \frac{y - 6}{x - 3} \]

\( PA \perp PB \rightarrow m_{PA} \times m_{PB} = -1 \)
\[ \frac{y - 2}{x + 1} \times \frac{y - 6}{x - 3} = -1 \]
\[ (y - 2)(y - 6) = -(x + 1)(x - 3) \]
\[ (x + 1)(x - 3) + (y - 2)(y - 6) = 0 \]
\[ x^2 - 2x - 3 + y^2 - 8y + 12 = 0 \]
\[ 2^2 - 2x + 1 + y^2 - 8y + 16 = 3 - 12 + 17 \]
\[ (x - 1)^2 + (y - 4)^2 = 8 \]

(ii) This is in the form of a circle with centre \((1, 4)\) and radius \(2\sqrt{2}\)

(c) Let \(d\) be the distance from the lines
\[ d = \frac{|13x + 4y|}{\sqrt{16 + 9}} \]
\[ = \frac{|3x + 4y|}{\sqrt{16 + 9}} \]
\[ 3x + 4y = 4x - 3y + 2 \quad \text{or} \quad 3x + 4y = -4x + 3y - 2 \]
\[ -x + 7y - 2 = 0 \quad \text{or} \quad 7x + y + 2 = 0 \]
\[ x - 7y + 2 = 0 \]

(d) This is a parabola of the form \(y^2 = 4a(y - k)\)
\( h = 0, \quad k = 1 \) and \( a = 1 \)
\[ y^2 = 4(y - 1) \text{ is the locus of } P \]